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LARGE FINANCIAL MARKETS: ASYMPTOTIC ARBITRAGE AND CONTIGUITY*

YU. M. KABANOV† AND D. O. KRAMKOV‡

(*Translated by Yu. M. Kabanov*)

Abstract. We introduce a large financial market as a sequence of ordinary security market models (in continuous or discrete time). An important property of such markets is the absence of asymptotic arbitrage, i.e., a possibility to obtain "essential" nonrisk profits from "infinitesimally" small endowments. It is shown that this property is closely related to the contiguity of the equivalent martingale measures. To check the "no asymptotic arbitrage" property one can use the criteria of contiguity based on the Hellinger processes. We give an example of a large market with correlated asset prices where the absence of asymptotic arbitrage forces the returns from the assets to approach the security market line of the CAPM.

Key words. large security market, no-arbitrage, equivalent martingale measure, contiguity of measures, Hellinger process, Capital Asset Pricing Model (CAPM)

1. Introduction. The fundamental theorem on asset prices by Harrison–Pliska [3] which gives a criterion for the absence of arbitrage in a finite discrete time security market has the following intuitive formulation: "If one cannot win betting on a process, then it must be a martingale under an equivalent measure" [1]. Thus, it provides a converse to the classical property "one cannot win betting on a martingale". Certainly, the well-known "St. Petersburg paradox" shows that the last statement is not absolute. Nevertheless, there are a number of other models where it still holds, possibly after a certain modification. We have a similar situation with the Harrison–Pliska theorem.

This theorem for a finite discrete time model in its natural formulation, i.e., without additional restrictions, has been proved comparatively recently by Dalang, Morton, and Willinger, [1] (for other proofs see [8], [11]). Continuous time models, as well as models with infinite horizon and/or infinite number of securities are now extensively studied by a number of authors (see, e.g., [2], [10], [11]). The problems turn out to be very delicate. One needs to modify the notion of "no-arbitrage" or "no free lunch" in order to obtain satisfactory theorems.

The present note also deals with such a modification. On an informal level one can think about a financial market with a "large" (infinite) number of traded securities. An investor is faced with the problem to choose a "reasonably large" number n of securities to make a self-financing portfolio. Starting from an initial endowment V_0^n, a trading strategy φ leads to the final value $V_T^n(\varphi)$ where the strategy φ and the time horizon T depend also on n. If an "infinitesimally" small endowment gives an "essential" gain with a positive probability (without any losses) we say that there exists an asymptotic arbitrage. To give a precise meaning to the above notions, it seems natural to consider as the model a sequence

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†Central Economics and Mathematics Institute, Krasikova st. 32, 117418 Moscow, Russia. This research was partially supported by International Science Foundation grant MMK000.

‡Steklov Mathematical Institute, Vavilova st. 42, 117966 Moscow, GSP-1, Russia. This research was partially supported by Russian Foundation of Fundamental Researches grant 93–011-1440 and by International Science Foundation grant MMK000.

of filtered probability spaces rather than a fixed one of the traditional theory. Such a device is of common use in mathematical statistics and we can apply the results of the latter in the present context.

At the end of the paper we give an example of a "large" market where the prices of securities are described by correlated diffusion processes. If these processes are geometric Brownian motions, the absence of asymptotic arbitrage is equivalent to a certain relation between the coefficients analogous to that arising in the classical Capital Asset Pricing Model (CAPM).

2. Asymptotic arbitrage and contiguity of martingale measures.

Let $\mathbf{B}^n = (\Omega^n, \mathcal{F}^n, \mathbf{F}^n = (\mathcal{F}^n_t), P^n)$, $n \in \mathbf{N}$, be a stochastic basis, i.e., a filtered probability space satisfying the usual assumptions (see, e.g., [5], [6]). For simplicity we assume that the initial σ-algebra is trivial (up to P^n-null sets). Asset prices evolve according to a semimartingale $S^n = (S^n_t)$ defined on $\mathbf{B}^n$ and taking values in $\mathbf{R}^d_+$ for some $d = d(n)$.

We fix a sequence T^n of positive numbers which are interpreted as time horizons. To simplify notations we shall often omit the superscript on T.

We shall call $(\mathbf{B}^n, S^n, T^n)$ a security market model and say that the sequence $\mathbf{M} = \{(\mathbf{B}^n, S^n, T^n)\}$ is a large security market.

It is assumed that there exists an asset the price of which is constant over time and all other prices are calculated in units of this asset. Markets are frictionless and admit shortselling.

Let P^n_T be the restriction of P^n to the σ-algebra $\mathcal{F}^n_T$.

We denote by $\mathcal{P}^n_T$ the set of all probability measures $\widetilde{P}^n_T$ equivalent to P^n_T and such that the process $(S^n_t)_{t \leqq T}$ is a local martingale with respect to $\widetilde{P}^n_T$. Of course, it may happen that $\mathcal{P}^n_T$ is empty. The existence of a measure $\widetilde{P}^n_T \in \mathcal{P}^n_T$ is closely related to the absence of arbitrage on the market $(\mathbf{B}^n, S^n, T^n)$ (see [1], [3], [10]–[13]), while the uniqueness is the property connected with the completeness of the market [3], [4].

Our main assumption is that the sets $\mathcal{P}^n_T$ are nonempty for all n.

Let $[S^n]$ be the quadratic variation of S^n, i.e., the process with values in the set of linear continuous maps from $\mathbf{R}^d$ into itself such that, for any $x \in \mathbf{R}^d$,

$$\Big([S^n]_t x, x\Big) = \Big\langle (S^n x, x)^c \Big\rangle_t + \sum_{s \leqq t} \Big(\Delta (S^n x, x)_s\Big)^2, \qquad t \leqq T,$$

where $(S^n x, x)^c$ is a continuous martingale part of $(S^n x, x)$, $\Delta X_s = X_s - X_{s-}$.

We define a trading strategy on $(\mathbf{B}^n, S^n, T^n)$ as a predictable process φ^n with values in $\mathbf{R}^d$ such that the process

$$\left(\int_0 I_{[0,T]}(t)\Big(d[S^n]_t \varphi^n_t, \varphi^n_t\Big)\right)^{1/2}$$

is locally integrable with respect to $\widetilde{P}^n_T$. For any trading strategy φ^n the stochastic integral

$$\varphi^n \cdot S^n_t = \int_0^t (\varphi^n_r, dS^n_r)$$

is well defined and does not depend on a particular choice of $\widetilde{P}^n_T$. The process $\varphi^n \cdot S^n$ is a local martingale with respect to $\widetilde{P}^n_T$ (and a semimartingale with respect to P^n_T) (see [5], [6]).

We shall consider on $(\mathbf{B}^n, S^n, T^n)$ also simple trading strategies which are predictable processes of the form

$$\varphi^n = \sum_{i=0}^k g_i I\Big(]\tau_i, \tau_{i+1}]\Big),$$

where g_i are $\mathcal{F}^n_{\tau_i}$-measurable random variables with values in $\mathbf{R}^d$, τ_i are stopping times with respect to $\mathbf{F}^n$ such that $0 = \tau_0 \leqq \tau_1 \leqq \ldots \leqq \tau_{n+1} = T$.

For a trading strategy φ^n and an initial endowment x^n a value process $V^n(\varphi^n)$ is given by

$$V_t^n(\varphi^n) = x^n + \varphi^n \cdot S_T^n = x^n + \int_0^t (\varphi_r^n, dS_r^n).$$

We shall include a positive number x^n (an initial endowment) in the definition of a trading strategy.

DEFINITION 1. A sequence of trading strategies φ^n realizes an asymptotic arbitrage of the first kind if

1a) $V_t^n(\varphi^n) \geqq 0$ for all $t \leqq T$;

1b) $\lim_n V_0^n(\varphi^n) = 0$ (i.e., $\lim_n x^n = 0$);

1c) $\lim_n P^n(V_T^n(\varphi^n) \geqq 1) > 0$.

DEFINITION 2. A sequence of trading strategies φ^n realizes an asymptotic arbitrage of the second kind if

2a) $V_t^n(\varphi^n) \leqq C$ for all $t \leqq T$ and some positive constant C;

2b) $\lim_n V_0^n(\varphi^n) > 0$;

2c) $\lim_n P^n(V_T^n(\varphi^n) \geqq \varepsilon) = 0$ for any $\varepsilon > 0$.

To achieve "almost nonrisk" profit from the arbitrage of the second kind, an investor sells short his portfolio. The constant C is interpreted as a bound for the total debt value.

DEFINITION 3. A large security market $\mathbf{M} = \{(\mathbf{B}^n, S^n, T^n)\}$ has no asymptotic arbitrage of the first kind (respectively, of the second kind) if for any subsequence n' there are no trading strategies $\varphi^{n'}$ realizing the asymptotic arbitrage of the first kind (respectively, of the second kind) for $\{(B^{n'}, S^{n'}, T^{n'})\}$.

If in the above definitions we admit only a simple trading strategy, then it is quite natural to use such terminology as "simple asymptotic arbitrage", and so on.

The following notion is well-known in mathematical statistics (see [6]).

DEFINITION 4. A sequence of probability measures $(\widetilde{P}_T^n)$ is contiguous with respect to (P_T^n) (write $(\widetilde{P}_T^n) \lhd (P_T^n)$) if for any sequence of sets $\Gamma^n \in \mathcal{F}_T^n$ with $P_T^n(\Gamma^n) \to 0$ we have $\widetilde{P}_T^n(\Gamma^n) \to 0$.

PROPOSITION 1. (a) *If $(P_T^n) \lhd (\widetilde{P}_T^n)$ for some sequence $\widetilde{P}_T^n \in \mathcal{P}_T^n$, then $\mathbf{M}$ has no asymptotic arbitrage of the first kind.*

(b) *If $(\widetilde{P}_T^n) \lhd (P_T^n)$ for some sequence $\widetilde{P}_T^n \in \mathcal{P}_T^n$, then $\mathbf{M}$ has no asymptotic arbitrage of the second kind.*

Proof. (a) Let φ^n be a sequence of trading strategies realizing the asymptotic arbitrage of the first kind. The process $V^n(\varphi^n)$ is a non-negative $\widetilde{P}^n$-local martingale, hence, a $\widetilde{P}^n$-supermartingale and

$$\widetilde{\mathbf{E}}^n V_T^n(\varphi^n) \leqq \widetilde{\mathbf{E}}^n V_0^n(\varphi^n) = x^n \to 0$$

by 1b). Thus, $\widetilde{P}^n(V_T^n(\varphi^n) \geqq 1) \to 0$ and, by virtue of contiguity $(P_T^n) \lhd (\widetilde{P}_T^n)$, it follows that $P_T^n(V^n(\varphi^n) \geqq 1) \to 0$ in contradiction with 1c).

(b) Let φ^n be a sequence of trading strategies φ^n realizing the asymptotic arbitrage of the second kind. In view of the contiguity $(\widetilde{P}_T^n) \lhd (P_T^n)$ it follows from 2c) that $\widetilde{P}^n(V_T^n(\varphi^n) \geqq \varepsilon) \to 0$ or, equivalently, $\widetilde{P}^n([V_T^n(\varphi^n)]^+ \geqq \varepsilon) \to 0$. Since $0 \leqq [V_T^n(\varphi^n)]^+ \leqq C$ this implies that $\widetilde{\mathbf{E}}^n[V_T^n(\varphi^n)]^+ \to 0$ as $n \to \infty$. The bounded process $[V^n(\varphi^n)]^+$ is a $\widetilde{P}^n$-submartingale. Thus,

$$\widetilde{\mathbf{E}}^n V_0^n(\varphi^n) \leqq \widetilde{\mathbf{E}}^n \Big[V_0^n(\varphi^n)\Big]^+ \leqq \widetilde{\mathbf{E}}^n \Big[V_T^n(\varphi^n)\Big]^+ \to 0$$

contradicting 2b).

PROPOSITION 2. *Assume that for any n the set $\mathcal{P}_T^n$ consists only of one point $\widetilde{P}_T^n$. Then,*

(a) *$(P_T^n) \lhd (\widetilde{P}_T^n)$ if and only if there is no asymptotic arbitrage of the first kind;*

(b) *$(\widetilde{P}_T^n) \lhd (P_T^n)$ if and only if there is no asymptotic arbitrage of the second kind.*

Proof. (a) Assume that (P_T^n) is not contiguous with respect to $(\widetilde{P}_T^n)$. Taking, if necessary, a subsequence we can find sets $\Gamma^n \in \mathcal{F}_T^n$ such that $\widetilde{P}^n(\Gamma^n) \to 0$, $P^n(\Gamma^n) \to \gamma > 0$ as

$n \to \infty$, where $\gamma > 0$. Let us consider the $\widetilde{P}^n$-martingale $X_t^n = \widetilde{\mathbf{E}}^n(I_{\Gamma^n} \mid \mathcal{F}_t^n)$. It follows from [5, Thm. 11.2] that there is a predictable process φ^n which is a trading strategy and

$$X^n = X_0^n + \varphi^n \cdot S^n = V^n(\varphi^n).$$

Since

$$V_0^n(\varphi^n) = \widetilde{\mathbf{E}}^n I_{\Gamma^n} = P^n(\Gamma^n) \to 0,$$

and

$$\lim_n P^n(V_T^n\big(\varphi^n) \geqq 1\big) = \lim_n P^n\Big(V_T^n(\varphi^n) = 1\Big) = \lim_n P^n(\Gamma^n) = \gamma > 0,$$

the trading strategy φ^n realizes the asymptotic arbitrage of the first kind.

(b) Assume that $(\widetilde{P}_T^n)$ is not contiguous with respect to (P_T^n). Taking, if necessary, a subsequence we can find sets $\Gamma^n \in \mathcal{F}_T^n$ such that $P^n(\Gamma^n) \to 0$, $\widetilde{P}^n(\Gamma^n) \to \gamma > 0$ as $n \to \infty$ for some $\gamma > 0$. Again let us consider the bounded $\widetilde{P}^n$-martingale $X_t^n = \widetilde{\mathbf{E}}^n(I_{\Gamma^n} \mid \mathcal{F}_t^n)$. Let φ^n be the trading strategy such that $X^n = X_0^n + \varphi^n \cdot S^n = V^n(\varphi^n)$. We have

$$V_0^n(\varphi^n) = V_0^n(\varphi^n) = \widetilde{\mathbf{E}}^n I_{\Gamma^n} = P^n(\Gamma^n) \to \gamma > 0,$$

$$\lim_n P^n\Big(V_T^n(\varphi^n) \geqq \varepsilon\Big) = \lim_n P^n(\Gamma^n) = 0.$$

Thus, φ^n gives the asymptotic arbitrage of the second kind.

If asset prices follow a continuous processes, then the above result can be strengthened.

PROPOSITION 3. *Assume that for any n the process S^n is continuous and the set $\mathcal{P}_T^n$ consists only of one point $\widetilde{P}_T^n$. Then,*

(a) $(P_T^n) \triangleleft (\widetilde{P}_T^n)$ *if and only if there is no simple asymptotic arbitrage of the first kind;*

(b) $(\widetilde{P}_T^n) \triangleleft (P_T^n)$ *if and only if there is no simple asymptotic arbitrage of the second kind.*

The proof follows immediately from Proposition 2 and the following lemma.

LEMMA 1. *Let $(\mathbf{B}, S, T)$ be a security market model. Assume that S is continuous and the set $\mathcal{P}_T$ is nonempty. Then for any trading strategy φ with value process $V(\varphi)$, $0 \leqq V(\varphi) \leqq 1$, there exists a simple trading strategy $\widetilde{\varphi}$ such that*

(i) $V_0(\widetilde{\varphi}) = V_0(\varphi)$;

(ii) $0 \leqq V(\widetilde{\varphi}) \leqq 1$;

(iii) $P(\mid V(\widetilde{\varphi}) - V(\varphi) \mid \geqq \varepsilon) \leqq \varepsilon$.

Proof. Let $\widetilde{P}$ be an element of $\mathcal{P}_T$. Since $V(\varphi)$ is bounded, it follows that

$$\widetilde{\mathbf{E}} \int_0^T \Big(d[S]_t \varphi_t, \varphi_t\Big) < \infty.$$

Approximating φ by predictable step functions in $L^2(d\widetilde{P}d[S])$, we find trading strategies φ^j with the same initial endowments $V_0(\varphi^j) = V_0(\varphi)$ such that $\widetilde{P}$-a.s (hence, P-a.s.)

$$\lim_{j \to \infty} \sup_{t \leqq T} \mid V_t(\varphi^j) - V_t(\varphi) \mid = 0.$$

By stopping at the exit time of $V(\varphi^j)$ from [0,1], we may assume that $0 \leqq V(\varphi^j) \leqq 1$ (due to continuity of S). Thus, we can choose a trading strategy $\widetilde{\varphi} = \varphi^j$ for some j which satisfies (i) and (ii) such that (iii) also holds.

The lemma is proved.

Remark. The assertion of Proposition 3, certainly, holds for the discrete time case (where all trading strategies are simple).

Propositions 1–3 relate the absence of asymptotic arbitrage to the problem of contiguity.

We recall now basic facts about contiguity of probability measures on a stochastic basis (for a detailed exposition see [6, Chap. IV, V]).

Let $(\Omega, \mathcal{F}, \mathbf{F})$ be a filtered space with probability measures P, $\widetilde{P}$ and $Q = (P + \widetilde{P})/2$. Assume that $\mathcal{F}$ is complete with respect to Q and (for simplicity) that $\mathcal{F}_0$ is generated by Q-null sets. Define local densities $z_t = dP_t/dQ_t$, $\widetilde{z}_t = d\widetilde{P}_t/dQ_t$. For $\alpha \in]0,1[$, let $Y_t(\alpha) = z_t^\alpha \widetilde{z}_t^{1-\alpha}$. The positive bounded process $Y_t(\alpha)$ is a Q-supermartingale. The Hellinger process $h_t(\alpha) = h_t(\alpha, P, \widetilde{P})$ is a predictable increasing process defined from the multiplicative decomposition $Y_t(\alpha) = M_t(\alpha)\mathcal{E}_t(-h(\alpha))$, where $M(\alpha)$ is a "local" martingale starting from zero, $\mathcal{E}(-h(\alpha))$ is the Doleans exponential, i.e., the solution of the linear equation

$$\mathcal{E}\Big(-h(\alpha)\Big) = 1 - \mathcal{E}_-\Big(-h(\alpha)\Big)\cdot h(\alpha).$$

The Hellinger process $h(\alpha)$ is uniquely determined up to the moment when the product $z\widetilde{z}$ hits zero.

The value $\alpha = \frac{1}{2}$ is usually omitted in the notations.

The Liptser–Shiryaev theorem gives the following criterion of contiguity:

$$(\widetilde{P}_T^n) \lhd (P_T^n) \quad \text{if and only if} \quad \lim_{\alpha\to 0} \limsup_{n\to\infty} \widetilde{P}^n\Big(h_T(\alpha, P^n, \widetilde{P}^n) \geqq \varepsilon\Big) = 0 \quad \text{for all } \varepsilon > 0.$$

In the case when the Hellinger process is continuous the criterion can be written in a simpler form:

$$(\widetilde{P}_T^n) \lhd (P_T^n) \quad \text{if and only if} \quad \lim_{K\to 0} \limsup_{n\to\infty} \widetilde{P}^n\Big(h_T(P^n, \widetilde{P}^n) \geqq K\Big) = 0.$$

For specific models the Hellinger processes can be calculated in terms of the semimartingale characteristics (see, e.g., [6], [7], [9]).

Example. Let $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t), P)$ be a stochastic basis with a countable set of independent standard one-dimensional Wiener processes w^i, $i \in \mathbf{Z}_+$, let $\mathbf{F}^n = (\mathcal{F}_t^n)$ be the filtration generated by w^i, $0 \leqq i \leqq n$. The behavior of stock prices is described by the following stochastic differential equations:

$$\begin{aligned} dX_t^0 &= \alpha_t^0 X_t^0 dt + \sigma_t^0 X_t^0 dw_t^0, \\ dX_t^i &= \alpha_t^i X_t^i dt + \sigma_t^i X_t^i \Big(\gamma_t^i dw_t^0 + \Big(1 - (\gamma_t^i)^2\Big)^{1/2} dw_t^i\Big), \qquad i \in \mathbf{N}, \end{aligned}$$

starting from some deterministic initial points. We assume that α^i and σ^i are scalar $\mathbf{F}^i$-predictable bounded processes, γ^i is a $\mathbf{F}^i$-predictable processes taking values in]-1,1[. To avoid technicalities we assume, moreover, that $|\sigma^i| \geqq c^i$ and $c^i \leqq |\gamma^i| \leqq 1 - c^i$ for some positive constant c^i.

Notice that the processes ξ^i with

$$d\xi_t^i = \gamma_t^i dw_t^0 + \Big(1 - (\gamma_t^i)^2\Big)^{1/2} dw_t^i,$$

are Wiener processes. The model reflects the fact that there are different kinds of randomness: proper to each stock itself and originated from some common source and accumulated in the stock "index" the evolution of which is described by the first equation.

Put $\beta^i = \gamma^i \sigma^i / \sigma^0$.

Let us consider the stochastic basis $\mathbf{B}^n = (\Omega, \mathcal{F}, \mathbf{F}^n = (\mathcal{F}_t^n), P)$ with the $(n+1)$-dimensional semimartingale $S^n = (X_t^0, X_t^1, \ldots, X_t^n)$. Let T^n be the time horizon. The sequence $\mathbf{M} = \{(\mathbf{B}^n, S^n, T^n)\}$ is a large security market with one-point sets $\mathcal{P}_T^n$. Under the measure $\widetilde{P}_T^n$ the processes $\widetilde{w}^0$, $\widetilde{w}^i$, $i \leqq n$, are Wiener processes with

$$\begin{aligned} d\widetilde{w}_t^0 &= \left(\frac{\alpha_t^0}{\sigma_t^0}\right) dt + dw_t^0, \\ d\widetilde{w}_t^i &= \Big(\alpha_t^i - \beta_t^i \alpha_t^0\Big)(\sigma_t^i)^{-1}\Big(1 - (\gamma_t^i)^2\Big)^{-1/2} dt + dw_t^i, \qquad t \leqq T. \end{aligned}$$

The Hellinger process $h_t^n = h_t(P_T^n, \widetilde{P}_T^n)$ is given by

$$8h_t^n = \int_0^t \left(\frac{\alpha_s^0}{\sigma_s^0}\right)^2 ds + \sum_{i=1}^n \int_0^t \left(\alpha_s^i - \beta_s^i \alpha_s^0\right)^2 (\sigma_s^i)^{-2} \left(1 - (\gamma_s^i)^2\right)^{-1} ds, \qquad t \leqq T,$$

(this follows from the general formula for the Hellinger process for multi-dimensional diffusion, see, e.g., [9, Thm. A2]).

PROPOSITION 4. (a) *There is no simple asymptotic arbitrage of the first kind if and only if*

$$\lim_{K\to\infty} \limsup_{n\to\infty} P^n(h_T^n \geqq K) = 0;$$

(b) *there is no simple asymptotic arbitrage of the second kind if and only if*

$$\lim_{K\to\infty} \limsup_{n\to\infty} \widetilde{P}^n(h_T^n \geqq K) = 0.$$

This assertion follows immediately from Proposition 3 and the criteria of contiguity.

Assume that all α^n, γ^n, and σ^n are deterministic, and T^n tends to $T^\infty > 0$. Then **M** has no simple asymptotic arbitrage of the first or/and the second kind if and only if

$$\int_0^{T^\infty} \left(\frac{\alpha_s^0}{\sigma_s^0}\right)^2 ds + \sum_{i=1}^\infty \int_0^{T^\infty} \left(\alpha_s^i - \beta_s^i \alpha_s^0\right)^2 (\sigma_s^i)^{-2} \left(1 - (\gamma_s^i)^2\right)^{-1} ds < \infty.$$

In the more specific case when all parameters are constant, $| \sigma^i | \leqq C$, $| \gamma^i | \leqq 1 - c$, and T^∞ is finite, we can see that **M** has no asymptotic arbitrage of the first and/or the second kind if and only if $\alpha^i = \beta^i \alpha^0 + u^i$, where $\sum (u^i)^2 < \infty$.

Notice that the measure of risk β^i has a clear economic interpretation. It is the covariance between the returns from the asset with number i and the index divided by the variance of the return from the index. The relation $\alpha^i = \beta^i \alpha^0$ is nothing but the security market line of the CAPM.

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